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**REDUCED MAGNETIC VECTOR
POTENTIAL AND ELECTRIC SCALAR
POTENTIAL FORMULATION FOR
EDDY CURRENT MODELING
(POSTPRINT)**



**Zhiwei Zeng, Xin Liu, Yiming Deng, Lalita Udpa, Jeremy S. Knopp,
and Gary Steffes**

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Reduced Magnetic Vector Potential and Electric Scalar Potential Formulation for Eddy Current Modeling

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Abstract: A new formulation using the reduced magnetic vector potential and the electric scalar potential is proposed for modeling of eddy current (EC) problems using finite element analysis. The major advantage of the proposed formulation is that it eliminates the need for meshing the current source. In EC nondestructive testing the probe coil scans across the sample which involves motion of the coil. Simulation of EC scans in conventional finite element models requires considerable re-meshing. In the proposed formulation, the mesh and system matrix remain unchanged with different coil positions. The incomplete factorization of the system matrix as pre-conditioner is performed only once during the entire scan, which results in a significant reduction of computation time.

Keywords: Reduced Magnetic Vector Potential, Eddy Current, Finite Element Method

1. Introduction

Finite element methods based on potentials have been successful in modeling eddy current (EC) problems. These formulations in general, require the discretization of the current source. However, in EC nondestructive testing (NDT) applications, the meshing of the EC probe excitation coil becomes very cumbersome particularly if the scanning motion of the probe coil is to be taken into account. A simple example is a coil above a conducting plate with a circular hole. If the coil and the hole are not coaxial, meshing becomes complex. As the coil scans the sample, re-meshing the coil at each scan position is necessary and very difficult. It has been shown that if the current source is not discretized accurately, the convergence of solution becomes slow [1]. Hence formulations that do not require meshing the current source are highly desirable. Such formulations offer many advantages as listed below:

- a) Mesh generation with complex coil shape becomes much easier because the current source is not meshed and is not integrated with other structures in the solution domain. The coil is discretized independently for numerical integration of analytical expressions of the field due to the source current.
- b) Convergence of the iterative solver is faster because the current source can be modeled accurately.

- c) Solution domain can be reduced because the secondary field generated by induced currents and/or magnetization is much smaller than the source field.
- d) Large coils, such as the saddle coil used in the magnetic resonance imaging, do not need to be meshed with the target structure, which again reduces the solution domain.
- e) It is easy to continuously vary coil parameters, such as liftoff and tilt angle, for parametric studies without changing mesh.
- f) Scanning of coil over a sample involved in a typical EC NDT process can be modeled without re-meshing. The computation error due to re-meshing is removed.
- g) Using appropriate formulation, as proposed in this paper, incomplete factorization of the system matrix, which is the most time-consuming process in the solution phase, is performed only once.

The $\mathbf{A}, V, \psi, \phi$ formulation employing the magnetic vector potential \mathbf{A} , the electric scalar potential V , the total magnetic scalar potential ψ , and the reduced magnetic scalar potential ϕ does not require meshing the current source. Unfortunately, this formulation is complex and more importantly it gives ill-conditioned system matrix in the presence of high-aspect-ratio elements and discontinuities in material properties [2]. A formulation using the magnetic vector potential \mathbf{A} , the reduced magnetic vector potential \mathbf{A}_r , and the electric scalar potential V is presented in [2]-[3]. In this paper, we propose a new formulation using only the reduced magnetic vector potential and the electric scalar potential. With this $\mathbf{A}_r, V, \mathbf{A}_r$ formulation, the system matrix remains the same despite changes in coil position. The incomplete factorization of the system matrix is performed only once, which results in significant savings in computation time.

2. Formulation

2.1 Magnetic Vector Potentials

The magnetic flux density \mathbf{B} can be decomposed into two parts, namely,

$$\mathbf{B} = \mathbf{B}_s + \mathbf{B}_r = \mu_0 \mathbf{H}_s + \mathbf{B}_r \quad (1)$$

where μ_0 is the permeability of air; \mathbf{B}_s and \mathbf{H}_s are the flux density and field intensity, respectively due to the excitation current density \mathbf{J}_s in free space and \mathbf{B}_r is the flux density due to induced currents and/or magnetization. It is important to note that these fields satisfy the conditions $\nabla \cdot \mathbf{B}_s = 0$ and $\nabla \times \mathbf{H}_s = \mathbf{J}_s$. Since $\nabla \cdot \mathbf{B}_r = \nabla \cdot \mathbf{B} - \nabla \cdot \mathbf{B}_s = 0$, \mathbf{B}_r can be written as curl of a vector, namely, $\mathbf{B}_r = \nabla \times \mathbf{A}_r$. \mathbf{A}_r is called the reduced magnetic vector potential. We can then express \mathbf{B} in terms of \mathbf{A}_r as

$$\mathbf{B} = \mu_0 \mathbf{H}_s + \nabla \times \mathbf{A}_r \quad (2)$$

where \mathbf{H}_s is calculated using Biot-Savart's law:

$$\mathbf{H}_s = \frac{1}{4\pi} \int_{\Omega'} \mathbf{J}_s(\mathbf{r}') \times \frac{\mathbf{r} - \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|^3} d\Omega' \quad (3)$$

In (3), Ω' is the volume of the current source; \mathbf{r} and \mathbf{r}' denote the coordinates of observation and source points, respectively. The commonly used magnetic vector potential \mathbf{A} is now called the total magnetic vector potential. \mathbf{A} can be decomposed as $\mathbf{A} = \mathbf{A}_r + \mathbf{A}_s$, where \mathbf{A}_s is the magnetic vector potential due to the source current in free space and is expressed as:

$$\mathbf{A}_s = \frac{\mu_0}{4\pi} \int_{\Omega'} \frac{\mathbf{J}_s(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d\Omega'. \quad (4)$$

The $\mathbf{A}, V\text{-}\mathbf{A}_r$ formulation is presented in [2]-[3], in which \mathbf{A} and V are used in conductors; \mathbf{A}_r is used in free space. The two vector potentials are then coupled on conductor surfaces using \mathbf{A}_s . In the next subsection, we propose the new $\mathbf{A}_r, V\text{-}\mathbf{A}_r$ formulation that does not use the total magnetic vector potential.

2.2 New $\mathbf{A}_r, V\text{-}\mathbf{A}_r$ Formulation

The new formulation stems from the classical $\mathbf{A}, V\text{-}\mathbf{A}$ formulation [4]:

$$\nabla \times \nu \nabla \times \mathbf{A} + j\omega\sigma\mathbf{A} + \sigma\nabla V = \mathbf{J}_s \quad (5)$$

$$\nabla \cdot (j\omega\sigma\mathbf{A} + \sigma\nabla V) = 0 \quad (6)$$

where ν , σ , and ω are the reluctivity, conductivity, and angular frequency, respectively. Equations (5)-(6) are used in the whole solution domain; whereas only (5) neglecting the eddy current term is used in nonconducting region. Substituting the decomposition of \mathbf{A} and the representation of \mathbf{J}_s by \mathbf{H}_s into (5)-(6), we have

$$\nabla \times \nu \nabla \times \mathbf{A}_r + j\omega\sigma\mathbf{A}_r + \sigma\nabla V = \nabla \times \mathbf{H}_s - \nabla \times \nu \nabla \times \mathbf{A}_s - j\omega\sigma\mathbf{A}_s \quad (7)$$

$$\nabla \cdot (-j\omega\sigma\mathbf{A}_r - \sigma\nabla V) = \nabla \cdot j\omega\sigma\mathbf{A}_s \quad (8)$$

Applying the Coulomb gauge on \mathbf{A}_r , i.e., $\nabla \cdot \mathbf{A}_r = 0$, and assuming constant ν within each element, (7) can be rewritten as

$$-\nu \nabla^2 \mathbf{A}_r + j\omega\sigma\mathbf{A}_r + \sigma\nabla V = (1 - \nu_r) \nabla \times \mathbf{H}_s - j\omega\sigma\mathbf{A}_s \quad (9)$$

where ν_r is the reciprocal of the relative permeability. Equations (9) and (8) are the governing equations of the $\mathbf{A}_r, V\text{-}\mathbf{A}_r$ formulation.

Note that the weak form of (8) is written as

$$\int_{\Omega} (\nabla N_i \cdot j\omega\sigma\mathbf{A}_r + \nabla N_i \cdot \sigma\nabla V) d\Omega + \oint_{\Gamma} N_i [-j\omega\sigma(\mathbf{A}_r + \mathbf{A}_s) - \sigma\nabla V] \cdot \hat{\mathbf{n}} d\Gamma = - \int_{\Omega} \nabla N_i \cdot j\omega\sigma\mathbf{A}_s d\Omega. \quad (10)$$

By neglecting the surface integral in (10), we implicitly set the normal component of induced currents on conductor surface to 0.

In contrast to the $\mathbf{A}, V\text{-}\mathbf{A}_r$ formulation, the advantage of the new formulation is that the total magnetic vector potential is not used. Therefore there is no need to account for the coupling of \mathbf{A} and \mathbf{A}_r on conductor surface. With the new formulation, the system matrix is the same for different coil positions. Hence the incomplete factorization of the system matrix as pre-conditioner is conducted only once during the entire scanning process. The cost of the new formulation is that besides \mathbf{H}_s , $\nabla \times \mathbf{H}_s$ in presence of magnetic materials and \mathbf{A}_s must also be evaluated. However, evaluating these analytical values takes much less time than that required for performing the incomplete factorization.

3. Results

The sample and defect geometry used in the simulation studies is illustrated in Fig. 1. The conducting plate has dimensions of 72 mm × 72 mm × 4 mm and conductivity of 2.741×10^7 S/m. A through hole of

radius 4 mm is placed at the center. A crack of dimensions $4 \text{ mm} \times 1.4 \text{ mm} \times 2 \text{ mm}$ is located either at the top or bottom of the plate. The coil having 100 turns and excited at 3 kHz has inner and outer radii of 4 mm and 6 mm, respectively and height of 2 mm. The liftoff distance between the bottom of the coil and the top of the plate is 1 mm. The coil scans across the hole in the x direction (the y coordinate of the coil center is always 0) with 1 mm step size parallel to the plate surface. The mesh in the x - y plane is shown in Fig. 2.

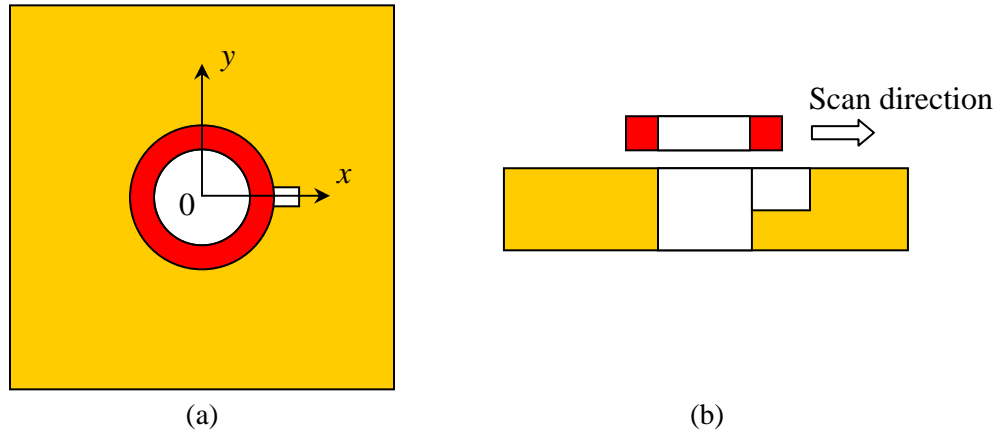


Fig. 1. Geometry investigated. (a) top view (b) side view.

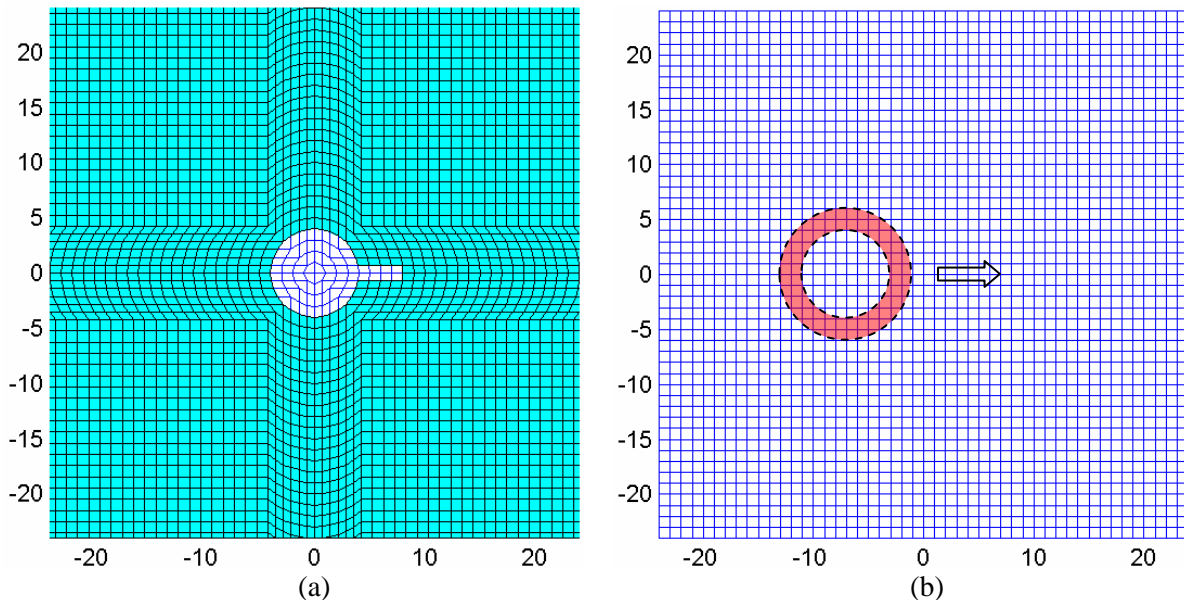


Fig. 2. Mesh in the x - y plane (units in mm). (a) in the plate with hole and crack (b) in air with coil.

The model is validated by comparing the model prediction of impedance of coil above the plate without hole and crack with the corresponding analytical value [5]. The analytical solution for coil impedance is $0.1641 + j1.8319 \Omega$ and the numerical value predicted by the model is $S_0 = 0.1648 + j1.7123 \Omega$. This demonstrates the validity of the new formulation.

The complex EC probe signal (coil impedance) as the probe scans the sample is calculated for the 3 cases: S_1 - sample with hole but no crack, S_2 - sample with hole and surface crack, and S_3 - sample with hole and subsurface crack. The changes in impedance due to the hole and crack, i.e., $S_1 - S_0$, $S_2 - S_0$, and $S_3 - S_0$ are plotted in Fig. 3. The effect of the surface crack is evidenced by the asymmetry of the signal.

Subsurface crack is also detectable in the phase plot. Computation results obtained using commercial software VIC-3D[®] are also shown in Fig. 3. Our results match well with VIC-3D[®] results.

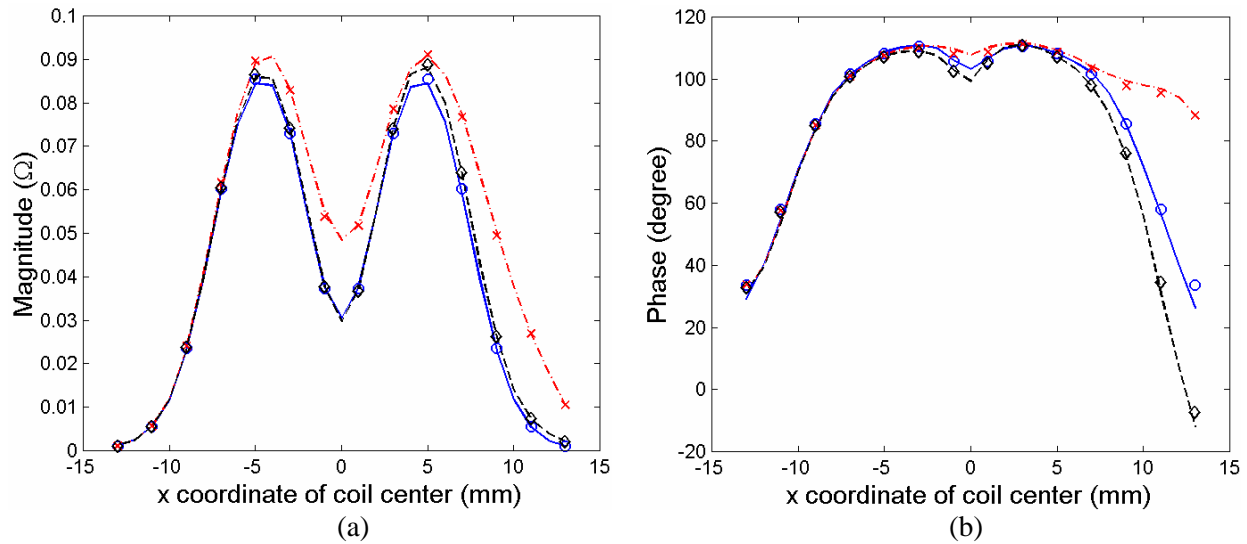


Fig. 3. Changes in EC probe impedance due to the hole and crack. (a) magnitude (b) phase.
New formulation: — hole; - - - hole and surface crack; - - - hole and subsurface crack.
VIC-3D[®]: ○ hole; × hole and surface crack ; ◇ hole and subsurface crack.

4. Conclusions

A new formulation using the reduced magnetic vector potential and the electric scalar potential for modeling EC problems is presented. Application of the formulation in NDT with coil scanning motion is studied. The major advantage of the formulation is in mesh generation since the coil is not required to be meshed at each position during the scan. Furthermore, the system matrix remains unchanged during the scanning process. This results in a significant reduction in computation time since the time-consuming incomplete factorization of the system matrix is performed only once. A more extensive testing of the model on multilayer structure is currently underway.

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